IB Calculus Assignment

Summer 2016

Mrs. White

Name: ____________________________

Grade (Circle One): 9 10 11 12

Calculator Used: __________________

Summer Assignment Grade: _________________

Please read carefully.

- *This packet is designed to review key Pre-Calculus IB topics. It is broken down into seven sections focused on in the IB Curriculum for Mathematics SL. These topics will be reviewed briefly at the beginning of the school year. You will be assessed on any/all of the topics focused on in this packet throughout the year.*

- *You are encouraged to complete this packet* ***on your own*** *with* ***minimal*** *outside help. It is intended to call your attention to any material you will need additional help on over the course of next year. You may use the IB Formulae packet. Please keep this packet handy and use it as reference throughout the 2016-2017 school year.*

- *An answer key will be posted on my website at the end of August.*

- ***YOU MUST HAVE THIS PACKET COMPELTED IN ENTIRETY ON THE FIRST DAY OF SCHOOL!*** *(****Wednesday, September 7, 2016*** *- the first day of class rotations). I will be grading it on your level of effort/completeness. We will go over any questions before you are assessed on the material. Pace yourself – It is lengthy! Try to complete one section every week and a half.*

- ***Also please be aware that your introduction and your data for your internal assessment paper will be due on Friday, September 30, 2016.*** *It would be wise to complete your introductory material over the summer. It will alleviate any unnecessary burdens on your workload as you begin your school year. I look forward to working with all of you on this endeavor!*

PART I: ALGEBRA

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1. Given**:** $f(x) = x^2 + 3x - 1$, find $\dfrac{f(x+\Delta x) - f(x)}{\Delta x}, \Delta x \neq 0$

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2. List all asymptotes for the following rational functions.

   a) $f(x) = \dfrac{x^2 + 5x + 8}{x + 3}$ b) $y = \dfrac{x - 3}{x^2 + 5x - 6}$ c) $y = \dfrac{x - 2}{x^2 - 3x + 2}$

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3. Determine whether the following functions are even, odd, or neither. Justify your answers algebraically, hence, validate by determining *f(-x).*

   a) $f(x) = -x^5 + 3x^3 - 2x + 1$ b) $g(x) = 5x^2 - 2$

   c) $h(x) = x^4 + 2x - 2$ d) $k(x) = x^7 - 4x$

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4. Determine any horizontal/vertical intercepts of $f(x) = 6x^3 - 19x^2 + 16x - 4$.

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5. Solve the given inequalities.

   a) $|3x + 1| \geq -4$ b) $x^2 + x - 1 \leq 5$

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6. Find the following sums

a) $\sum_{n=1}^{6}\left(n^2 - n\right)$ b) $\sum_{n=1}^{4}\left(2 + \frac{5}{2}n - \frac{3}{2}n^2\right)$

7. Solve the equation: $9^{x-1} = \left(\frac{1}{3}\right)^{2x}$

8. Let $\log_{10} P = x$, $\log_{10} Q = y$, and $\log_{10} R = z$. Express $\log_{10}\left(\frac{P}{QR^3}\right)^2$ in terms of x, y, and z.

9. Solve the equation: $\log_{27} x = 1 - \log_{27}(x - 0.4)$.

10. A sum of $5,000 is invested at a continuously compounded interest rate of 6.3% per annum.

a) Write down an expression for the value of the investment after n full years.

b) What will be the value of the investment at the end of five years?

c) The value of the investment will exceed $10,000 after *n* full years. Calculate the minimum value of *n*.

11. Find the coefficient of x^5 in the expansion of $(3x-2)^8$.

12. Determine an expression to represent the given sequences below. Then find a formula to find the sum of the first n terms. (Assume n = 1 represents the first term).

a. 25, 22, 19, 16, ….

b. $1, \frac{9}{10}, \frac{81}{100}, \frac{729}{1000}, \ldots.$

c. $\frac{1}{(2)(3)}, \frac{1}{(3)(4)}, \frac{1}{(4)(5)}, \frac{1}{(5)(6)}, \ldots.$

13. Find the sum of the infinite geometric series: $\frac{2}{3} - \frac{4}{9} + \frac{8}{27} - \frac{16}{81} \ldots$

14. The first three terms of an arithmetic sequence are 7, 9.5, 12. What is the sum of the first 101 terms of the sequence?

15. In an arithmetic sequence, the first term is -2, the fourth term is 16 and the n^{th} term is 11,998. Find the common difference, *d*, and the value of *n*.

PART II: FUNCTIONS

1. Let $f(x) = e^{-x}$ and $g(x) = \dfrac{x}{1+x}$, $x \neq -1$. Find the following <u>and</u> the domain of each.

    a) $f^{-1}(x)$ b) $(f \circ g)(x)$ c) $g(f(x))$

    d) $(f \cdot g)(x)$ e) $\left(\dfrac{f}{g}\right)(x)$ f) $f(g(0))$

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2. The equation $kx^2 + 3x + 1 = 0$ has exactly one solution. Find the value of k.

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3. The mass *m* kg of a radio-active substance at time t hours is given by $m = 4e^{-0.2t}$. If the mass is reduced to 1.5 kg. How long does it take?

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4. The function *f* is given by $f(x) = x^2 - 6x + 13$ for $x \geq 3$.

    a) Write $f(x)$ in the form of $\left(x - a^2\right) + b$.

    b) Find the inverse function of $f(x)$.

    c) State the domain of the inverse.

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5. The equation $x^2 - 2kx + 1 = 0$ has two distinct real roots. Find the set of all possible values of *k*.

6. The functions f and g are defined by $f :\mapsto 3x$ and $g :\mapsto x+2$.

   a) Find an expression for $(f \circ g)(x)$.

   b) Find the value of $f^{-1}(x)+g^{-1}(x)$ when x equals 17.

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7. The quadratic function *f* is defined by $f(x)=3x^2-12x+11$.

   a) Write *f* in the form of $f(x)=3(x-h)^2-k$.

   b) The graph of *f* is translated 5 units in the negative x-direction and 3 units the negative y-direction. Find the function g for the translated graph, given your answer in the form of $g(x)=3(x-p)^2+q$.

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8. The function f is given by $f(x)=e^{(x-11)}-8$. Find $f^{-1}(x)$ <u>and</u> its domain.

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9. The point *A* (3, -1) is on the graph of *f*. The point A' is the corresponding point on the graph of $y=-2f(x)+4$. Find the coordinates of A'.

PART III: TRIGONOMETRY

1. Find the x-coordinates of the solution(s) of $2+\cos(2x)-2\sin(0.5x)=0$ for $0\le x\le 3$, where x is in radians.

2. Given $f(x)=3\sin^2 x-11\sin x+6$.
   a) Factorize the expression.

   b) Find the two values of sin x which satisfy the equation

   c) Solve the equation for $0^\circ \le x \le 180^\circ$

3. In a triangle ABC, AB = 4cm, AC = 3cm and the area of the triangle is 4.5cm^2. Find the two possible values of the angle BAC.

4. Solve the equation $2\cos^2(x)=\sin(2x)$ for $0\le x\le \pi$ giving your answers in terms of π.

5. In triangle ABC, BC = 5cm, angle B is 60 degrees, and angle C is 40 degrees.
   a) Find AB.

   b) Find the area of the triangle.

6. Consider $y = \sin\left(x + \frac{\pi}{9}\right)$

a) The graph of y intersects the x-axis at point A. Find the x-coordinate of A, where $0 \le x \le \pi$.

b) Solve the equation $-\frac{1}{2} = \sin\left(x + \frac{\pi}{9}\right)$, for $0 \le x \le 2\pi$

7. Verify the identity: $\frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta} = 2\sec^2\theta$

8. Use the properties of logarithms and trigonometric identities to verify this identity: $-\ln(1+\cos\theta) = \ln(1-\cos\theta) - 2\ln|\sin\theta|$

9. Solve: $3\tan^2\theta - 1 = 0$

10. Solve: $2\sin^2 x + 3\cos x - 3 = 0$

PART IV: VECTORS

1. ABCD is a rectangle and O is the midpoint of AB.
    a) Create a drawing representing the problem.

    b) Express each of the following vectors in terms of $\overrightarrow{OC}$ and $\overrightarrow{OD}$.
        a) $\overrightarrow{CD}$ b) $\overrightarrow{OA}$ c) $\overrightarrow{AD}$

2. The vectors *i* and *j* are unit vectors along the x-axis and y-axis respectively.
    a) Given the vectors $u = -i + j$ and $v = 3i + 5j$, find $u + 2v$ in terms of *i* and *j*.

    b) A vector *w* has the same direction as $u + 2v$, and has a magnitude of 26. Find *w* in terms of *i* and *j*.

3. A line passes through the point (4, -1) and its direction is perpendicular to the vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$. Find the equation of the line in the form $ax + by = p$ where *a, b,* and *p* are integers to be determined.

4. The vectors $\begin{pmatrix} 2x \\ x-3 \end{pmatrix}$ and $\begin{pmatrix} x+1 \\ 5 \end{pmatrix}$ are orthogonal for two values of x. Find the two values of x.

5. A balloon is moving at a constant height with a speed of 18kmh^{-1} in the direction of $\begin{pmatrix}3\\4\\0\end{pmatrix}$. At time $t = 0$, the balloon is at point B with coordinates $(0, 0, 5)$.

a) Show that the position vector b of the balloon at time t is given by $b = \begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}0\\0\\5\end{pmatrix} + t\begin{pmatrix}10.8\\14.4\\0\end{pmatrix}$

At time $t = 0$, a helicopter goes to deliver a message to the balloon. The position vector h of the helicopter at time t is given by $h = \begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}49\\32\\0\end{pmatrix} + t\begin{pmatrix}-48\\-24\\6\end{pmatrix}$

b) Write down the coordinates of the starting position of the helicopter.

c) Find the speed of the helicopter.

The helicopter reaches the balloon at point R.

d) Find the time the helicopter takes to reach the balloon.

e) Find the coordinates of R.

PART V: PROBABILITY

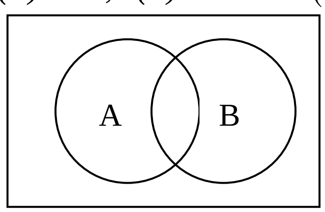
1. In a survey of 200 people, 90 of whom were female, it was found that 60 people were unemployed, including 20 males.
   a) Use the information to complete the table below.

	Male	**Female**	**Totals**
Unemployed			
Employed			
Totals			**200**

   b) If a person is selected at random from this group of 200, find the probability that this person is
      1. An unemployed female

      2. A male, given that the person is employed.

2. The following Venn diagram shows a sample space U and events A and B where $n(U) = 36$, $n(A) = 11$, $n(B) = 6$ and $n(A \cup B)' = 21$.



   a) On the diagram, shade the region $(A \cup B)'$.

   b) Find $n(A \cap B)$.

   c) Find $p(A \cap B)$.

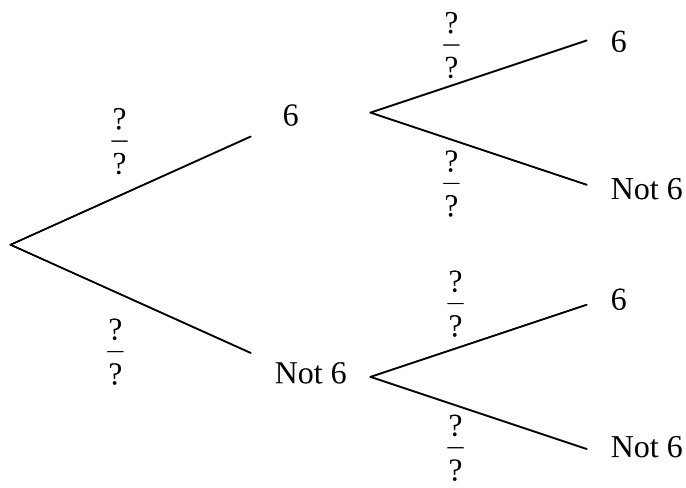
   d) Determine if the events are mutually exclusive. Explain your answer.

3. For the events A and B, $p(A) = 0.6$, $p(B) = 0.8$ and $p(A \cup B) = 1$. Find

   a) $p(A \cap B)$

   b) $p(A' \cup B')$

4. Two fair 6-sided dice are rolled and the total score is noted.
   a) Complete the tree diagram by entering the probabilities.



   b) Find the probability of getting one or more sixes.

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5. A bag contains 10 red chips, 10 green chips and 6 white chips. Two chips are drawn at random from the bag without replacement. What is the probability that they are of different colors?

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6. A factory makes staplers. Over a long period of time 4% of them are faulty. A random sample of 200 staplers is tested.
   a) Write down the expected number of faulty staplers in the sample.

   b) Find the probability that exactly 5 calculators are faulty.

   c) Find the probability that one or more are faulty.

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7. A family of functions is given by $f(x) = x^2 + 3x + k$, where $k \in \{1,2,3,4,5,6,7\}$. One of these functions is chosen at random. Calculate the probability that the curve of this function crosses the x-axis.

PART VI: STATISTICS

1. Given the following frequency distribution find the value of the mean and the median.

Number (x)	1	2	3	4	5	6
Frequency (f)	5	9	16	18	20	7

Mean = ________ Median = ________

2. From January to September, the mean number of car accidents per month was 630. From October to December, the mean was 810 accidents per month. What was the mean number of car accidents per month for the whole year?

3. Let a, b, c and d be integers such that $a < b < c$ and $c = d$. The mode of these four numbers is 11. The range of these four numbers is 8. The mean of these four numbers is 8. Find the values of a, b, c and d.

4. The 45 students in a class each recorded the number of whole minutes, x, spent doing experiments on Monday. The results are $\sum x = 2230$.
   a) Find the mean number of minutes the students spent on doing experiments on Monday.
   b) Two new students joined the class and reported that they spent 37 and 30 minutes respectively. Calculate the new mean for the class.

5. The numbers of hours of sleep of 21 students are shown in the frequency table below.

Hours of Sleep	**Number of Students**
4	2
5	5
6	4
7	3
8	4
10	2
12	1

Find the

a) Mean

b) Median

c) Lower Quartile

d) Upper Quartile

e) Interquartile Range

f) Mode

6. The table below shows the marks gained in a test by a group of students.

Mark	1	2	3	4	5
# of Students	5	10	p	6	2

The median is 3 and the mode is 2. Find the **two** possible values of p.

PART VII: LIMITS

1. Find the following limits. State the process you used to find the value (Substitution, graphing, rationalization, etc). If the limit doesn't exist, please state the reason why.

a) $\lim_{x \to 2}\left(x^2 + 3x - 4\right)$

b) $\lim_{x \to 8}\dfrac{\sqrt{x+1}}{x-4}$

c) $\lim_{x \uparrow \frac{1}{4}}\dfrac{\tan \pi x}{2}$

d) $\lim_{x \to 4}\dfrac{\sqrt{x+5}-3}{x-4}$

e) $\lim_{x \to 0}\cos\dfrac{1}{x}$

f) $\lim_{x \to 1}\arccos\dfrac{x}{2}$

g) $\lim_{t \to -2}\dfrac{t^3+8}{t+2}$

h) $\lim_{x \to 0}\dfrac{\frac{1}{5-x}-\frac{1}{5}}{x}$

i) $\lim_{x \to 2}\dfrac{4-\sqrt{18-x}}{x-2}$